Some impt. definitions 24/02/2026

1. ϵ-net

Let (X, d) be a metric space and $\epsilon > 0$ be a positive real number. Let A be a finite subset of X. Then A is called an ϵ-net for X iff for every $x \in X$, there exists a point $a \in A$ such that

$$d(a, x) < \epsilon$$

$\because d(a, x) < \epsilon \Rightarrow x \in S(a, \epsilon) \; \forall a \in A$.

Hence the finite subset A is an ϵ-net for X iff A is finite and

$$X = \cup \{ S(a, \epsilon) : a \in A \}$$

2. Totally bounded metric space

A metric space (X, d) is called totally bounded if and only if

X has an ϵ-net for every $\epsilon > 0$.